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A note on r-circulant matrices involving generalized Narayana numbers. (English)

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This article is about the relation between structured matrices and integer sequences. The authors [Appl. Math. Comput. 437, Article ID 127521, 13 p. (2023; Zbl 1510.11059)] already considered r-circulant matrices containing Narayana numbers to examine this relationship. Here they obtain estimates for spectral norms and eigenvalues of r-circulant matrices with entries from a Fibonacci-Narayana sequence, thus providing, for example, conditions for invertibility. There are many applications in physics, engineering, economics, signal theory and image processing for this kind of results.

The present paper starts with the Narayana sequence $\mathcal{N}_n = \mathcal{N}_{n-1} + \mathcal{N}_{n-3}, n \geq 3$, where $\mathcal{N}_0 = 0, \mathcal{N}_1 = 1$ and $\mathcal{N}_2 = 1$, which is closely related to the Fibonacci sequence $f_n = f_{n-1} + f_{n-2}, n \geq 2$, where $f_0 = 0, f_1 = 1$. A matrix $C = \mathrm{Circ}_r(u)$ in $\mathbb{M}^n(\mathbb{C})$ with entries

$$c_{ij} = \begin{cases} cj - i, & j \geq i \\ rc_{(n+j-i)} & j < i \end{cases}$$

is called r-circulant. Here $n \geq 2$ is an integer, $u = (c_0, c_1, \ldots, c_{(n-1)})^T \in \mathbb{C}^n$, and $r \in \mathbb{C}^n$. The main object of interest in the paper is given by r-circulant matrices

$$\mathrm{Circ}_r(\mathcal{N}_0, \mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{n-1}).$$

Eigenvalues and eigenvectors for such matrices are calculated and singularity and regularity conditions are given.

The following result is given in addition to the conclusions:

Corollary. The number of factors of matrix $\mathrm{Circ}_{-1}(\mathcal{N}_0, \mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_{n-1})$ is infinite.

The proof can be found in the paper [Aligarh Bull. Math. 41, No. 1, 41–52 (2022; Zbl 1524.15041)] by the reviewer.

Reviewer: Hasan Keleş (Trabzon)

MSC:

15B36 Matrices of integers
15B05 Toeplitz, Cauchy, and related matrices
11C20 Matrices, determinants in number theory
15A18 Eigenvalues, singular values, and eigenvectors
15A60 Norms of matrices, numerical range, applications of functional analysis to matrix theory

Keywords:

Narayana's cows sequence; r-circulant matrix; Euclidean norm; spectral norm; eigenvalues

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