4th International Civil Engineering & Architecture Conference
17-19 May 2025, Trabzon, Türkiye

Comparison of integrated force method and displacement method for metaheuristic optimization of civil structural systems

Ibrahim Torunoglu[1], Ayse T. Daloglu[*1]

[1] *Karadeniz Technical University, Department of Civil Engineering, 61080 Trabzon, Türkiye*

Abstract. In the optimization process where metaheuristic methods are used, large number of structural analysis is needed due to the nature of the methods. The most popular method used in structural analysis in civil engineering is the finite element displacement method. In the displacement method, after the global stiffness matrix is created, first the nodal displacements and then the internal forces of the elements are calculated. And these steps are repeated in each iteration, depending on the optimization method used, and naturally requires longer time for the analysis. Especially in structural systems with large number of elements, reducing analysis times can be an advantageous. In order to benefit from this advantage, choosing different structural analysis methods appears as an alternative. In this context, the performance of metaheuristic optimization methods combined with Integrated Force Method for structural analysis have been investigated. In the Integrated Force Method, first the internal forces of the elements and then the displacements of the nodes are calculated. Displacement method and Integrated Force Method are compared in terms of section optimization based on Harmony Search Algorithm and Teaching Learning Based Optimization. The obtained data show that the CPU times spent in the optimization processes using Integrated Force Method are shorter than the times spent in the case of Displacement Method.

Keywords: Finite Element Analysis; Integrated Force Method; Displacement Method; Metaheuristic Optimization

1. Introduction

From a structural engineering perspective, structural design can be considered as two stages. The first stage can be described as structural analysis and the second stage as structural design. Both stages are intertwined with each other. Pre-design is needed to perform structural analyses, and structural analysis results are needed to perform design.

In parallel with the ever-increasing vital needs of mankind, the developing engineering science has developed and continues to develop new methods for the structural analysis of increasingly complex physical models. Analytical methods, which began to be inadequate in solving complex structural models, began to give way to numerical methods as of the second half of the 20th century with the development of computer technology. The leading numerical method is the finite element method. The finite element method is divided into two separate methods. The first of these is the finite element displacement method, which is actively used in the analysis of today's engineering models, and the second is the finite element force method, which has remained in the background compared to the first. In both methods, the structural model, which is a continuum, is addressed by discretizing it into finite parts, and is analyzed under external loads by taking into account the equilibrium, boundary and compatibility conditions. The fundamental difference between the two methods is the assumption of unknowns. In the displacement method, the main unknowns are the displacements and rotations at the nodes. The internal forces occurring in the discrete elements are obtained with secondary calculations. In the force method, the main unknowns are the internal forces occurring in the elements (axial, shear, moment etc.). In this case, the displacements and rotations occurring at the nodes of the discrete elements are obtained by secondary calculations.

Increasing construction costs on a global scale, decrease in raw materials, and aesthetic concerns due to architectural reasons force the selection of cross sections to be minimized. However, earthquakes and wind loads, which have a significant effect especially on steel structures, force the designer to increase the section. The most important design criterions that comes out of this are; aesthetics, cost, sufficient strength, rigidity and ductility. Considering the factors that force the cross section selection in the minimum and maximum direction, reaching the optimum has become inevitable. This type of cross section optimization can be done manually by trial and

[*] Corresponding author, E-mail: aysed@ktu.edu.tr

https://doi.org/10.31462/icearc2025_ce_sme_363

error and in a limited number of times. With the development of computer technology, cross section optimizations have now reached programmable levels simultaneously with structural analysis and design. Structural engineers have to design structures with minimum weight in order to choose the optimum one among the applicable designs (Artar, 2015). Considering steel structure optimization only in terms of bearing capacity and servisibilty constraints is insufficient in practice. Secondary constraints are regulations, specifications, standards, manufacturability and usable material section types that may vary from country to country. In this case, a very wide solution space appears for the optimization of steel structures. In this case, optimization methods that use mathematical programming techniques may not be reliable in terms of optimal convergence in such solution spaces (Hasançebi & Saka, 2010). Naturally, meta-heuristic methods have begun to be used to obtain optimum solutions in such wide solution spaces (Artar & Daloğlu, 2018). In case meta-heuristic methods are preferred, sometimes hundreds or even tens of thousands of structural analyzes need to be performed even on the smallest structural models. This circumstance results in a time constraint for the designer. In order to compare the length of the central processing unit (CPU) operating time in the case of using metaheuristic methods in the structural analysis and dimensioning process, the finite element displacement method (DM) and the finite element integrated force method (IFM) were compared. Harmony Search Algorithm (HSA) and Teaching-Learning Based Optimization (TLBO) methods were preferred as optimization techniques.

2. Integrated force method

The integrated force method was first proposed by Patnaik (1973). IFM, as in the finite element displacement method (DM), is based on the discretization of the continuum of the structural model into finite parts. Unlike DM, the force equilibriums on the nodes are taken into account. It consists of the solution of the linear equation system obtained by creating the equilibrium equations of the internal forces (moment, shear, axial, etc.) and external loads belonging to the elements which connected at the nodes. The "mxn" dimensional "B" matrix shown in Equation (1) represents the equilibrium matrix. The "nx1" dimensional "F" vector represents the internal forces belonging to the elements, and the "mx1" dimensional "P" vector represents the external forces acting on the nodes. The "m" value in the equation represents the total number of equilibrium equations, and "n" represents the number of unknown internal forces. If "m=n", the system is defined as statically determined, and if "m<n", it is defined as statically undetermined (Patnaik, 1973).

$$[B]_{mxn}|F|_{nx1} = |P|_{mx1} \tag{1}$$

Instead of creating the equilibrium equations for each node one by one, calculating the local equilibrium matrices of the elements provides convenience in terms of numerical operations (Topçu, 2017). By calculating the local equilibrium matrix "$\hat{B}$" separately for each element, the global equilibrium matrix "B_e" of the element and the global equilibrium matrix "B" of the system can be calculated with the help of the transformation matrix. For example, in Fig. 1, the local end forces at the ends "i" and "j" of a plane truss "k."th element are shown, and in Fig. 2, the global end forces at the ends "i" and "j" are shown.

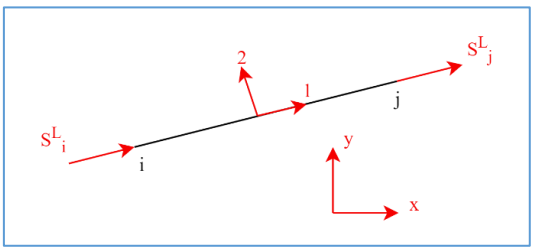


Fig. 1. Local internal forces of a plane truss element

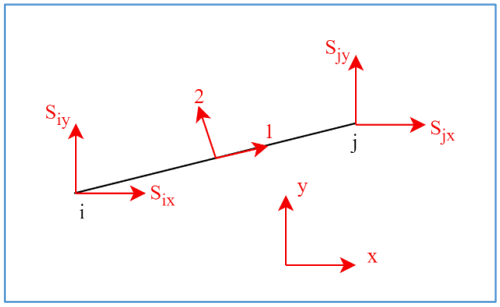


Fig. 2. Global internal forces of a plane truss element

The steps of calculating the local and global equilibrium matrices of the "k."th element in Fig. 1 and Fig. 2 are shown in eq. (2). In plane truss elements, only the axial internal force "F" occurs. The local and global end forces of the truss element can be expressed in matrix form in terms of the axial internal force.

$$S_j^L = F_k \Rightarrow \underset{\hat{S}}{\begin{vmatrix} \hat{S}_i^L \\ \hat{S}_j^L \end{vmatrix}} = \underset{\hat{B}}{\begin{vmatrix} -1 \\ 1 \end{vmatrix}} F_k$$

$$\underset{S}{\begin{vmatrix} S_{ix} \\ S_{iy} \\ S_{jx} \\ S_{jy} \end{vmatrix}} = \underbrace{\begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix}}_{T^T} \underset{\hat{S}}{\begin{vmatrix} \hat{S}_i^L \\ \hat{S}_j^L \end{vmatrix}} \quad (\mathrm{T;transformation\ matrix})$$

$$\underset{S}{\begin{vmatrix} S_{ix} \\ S_{iy} \\ S_{jx} \\ S_{jy} \end{vmatrix}} = \underbrace{\begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix}}_{T^T} \underset{\hat{B}}{\begin{vmatrix} -1 \\ 1 \end{vmatrix}} F_k \tag{2}$$

$$\underset{S}{\begin{vmatrix} S_{ix} \\ S_{iy} \\ S_{jx} \\ S_{jy} \end{vmatrix}} = \underset{B_e}{\begin{vmatrix} -c \\ -s \\ c \\ s \end{vmatrix}} F_k$$

In case the structural system is statically determined (m=n), eq. (1) can be solved by direct methods. In case the structural system is statically undetermined (n>m), (r=n-m) additional equations are needed. "r" represents the degree of hyperstaticity of the system. Additional equations are obtained by equating the work of the internal and external forces (eq. (3)). In eq. (3), "β" and "U" vectors represent the deformation vector of the elements and displacement vector of the nodes, respectively.

$$\frac{1}{2}\left|F^T\right|\left|\beta\right| = \frac{1}{2}\left|P^T\right|\left|U\right| \tag{3}$$

If the expression in eq. (1) is substituted into eq. (3), the expression in eq. (4) is obtained (Deformation-Displacement Relationship, DDR). Thus, with the help of eq. (4), "n" number of deformations are expressed with "m" number of displacements.

$$\left|\beta\right| = \left[B\right]^T \left|U\right| \tag{4}$$

If the displacements are expressed in terms of deformations, eq. (5) is obtained. In this expression, the matrix "C" is the "rxn" dimensional compatibility condition (CC). CC can be obtained by the SVD method.

$$\left[C\right]\left|\beta\right| = \left|0\right| \tag{5}$$

If the deformation expression in eq. (5) is expressed in terms of internal forces for each element and written in matrix form, eq. (6) which expressing the Force-Deformation Relationship (FDR) is obtained.

$$\beta_k = \underset{G_k}{\frac{l}{AE}} F_k \Rightarrow \begin{vmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{vmatrix} = \underbrace{\begin{bmatrix} G_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & G_n \end{bmatrix}}_{G} \begin{vmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{vmatrix} \tag{6}$$

The "G" matrix expressed in eq. (6) is the concatenated flexibility matrix of the structure. If eq. (6) is substituted into eq. (5), eq. (7) is obtained. Eq. (7) has the dimension "rxn" and the unknown vector is expressed in terms of internal forces. In this case, eq. (1) and (7) can be combined into a single equation system.

$$\left[C\right]\left[G\right]\left|F\right| = \left|0\right| \tag{7}$$

$$\begin{bmatrix} & \left[B\right] & \\ \left[C\right] & & \left[G\right] \end{bmatrix} \left|F\right| = \begin{vmatrix} P \\ 0 \end{vmatrix} \Leftrightarrow \quad \left[S\right]\left|F\right| = \left|P\right|^* \tag{8}$$

The coefficient matrix "S" in eq (8) is "nxn" dimension and the internal forces "F" can be obtained by direct methods. Nodal displacements are calculated with the help of eq. (9).

$$|U| = [J][G]|F| \tag{9}$$

The "J" matrix witch expressed in eq. (9) is;

$$[J] = m\,rows\,of\left[[S]^{-1}\right]^{T} \tag{10}$$

3. Optimization techniques used in this study

In this study TLBO and HSA methods are used to compare the integrated force method and the displacement method in aspect of CPU time. Before giving theoretical information about the methods to be used, the objective function and penalty coefficients used in almost every structural optimization process will be briefly stated.

Considering that the main purpose of structural optimization is to reduce the structural weight, it is inevitable to adopt the structural weight as the objective function.The objective function to be minimized for these methods is expressed in eq. (11).

$$\min.W = \sum_{k=1}^{ng} A_k \sum_{i=1}^{nk} \rho_i L_i \tag{11}$$

The penalty coefficient calculation method used for element cross sections and node displacements is taken into account as shown in eq. (12). The expression "C" is the penalty coefficient of the entire system, which includes the sum of the element and node penalty coefficients.

$$c_j = \frac{\sigma_j - \sigma_{u/l}}{\sigma_{u/l}},\ c_i = \frac{\delta_i - \delta_{u/l}}{\delta_{u/l}},\ C = \sum_{j=1}^{m} c_j + \sum_{i=1}^{n} c_i \tag{12}$$

The penalized objective function expresses the circumstance where the objective function is increased by the penalty coefficient. The expression "ε" in eq. (13) is the positive valued penalty multiplier and is taken into account as "2" in the current study.

$$\psi(x) = W(x)(1+C)^{\varepsilon} \tag{13}$$

3.1. Teaching learning based optimization

Like many metaheuristic optimization methods, TLBO is also a population-based optimization method and consists of populations of candidate solution sets to obtain the optimum solution (Rao et al., 2011). TLBO method is based on the principle that a good teacher will positively affect the grades (design variables) of students in the class (solution set). Similarly, the interaction of students with each other (information exchange) will positively affect their grades (Rao et al., 2011).

The TLBO method consists of two stages. The first stage is the teacher, and the second stage is the student stage (Rao et al., 2011). At the teacher stage, the student with the best level of knowledge (the individual with the best objective function) in the class (population) increases the average knowledge of the class by transferring his knowledge to the whole class (Rao et al., 2011). In the second stage, each student interacts with other students, exchanges information with each other and increases their knowledge levels (Rao et al., 2011).

Since TLBO is a population-based optimization method, firstly each student in the population is created with randomly selected values from the solution space. As can be seen from the relation in eq. (14), each student represents a solution set and the number of elements of the set (X^i) is equal to the number of design variables (D).

$$\begin{aligned} X^i &= \left[x_1^i\ x_2^i\ x_3^i\ \ldots x_j^i\right] \\ i &\in (1,2,3,\ldots NP) \\ j &\in (1,2,3,\ldots D) \end{aligned} \tag{14}$$

The students of the class ($X^{1,2,\ldots,NP}$) and their current knowledge levels ($\psi(x^{1,2,\ldots,NP})$) are shown in eq. (15).

$$\text{class} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_D^1 \\ x_1^2 & x_2^2 & \cdots & x_D^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{NP} & x_2^{NP} & \cdots & x_D^{NP} \end{bmatrix} \begin{matrix} \rightarrow \psi(x^1) \\ \rightarrow \psi(x^2) \\ \rightarrow \ \vdots \\ \rightarrow \psi(x^{NP}) \end{matrix} \tag{15}$$

In the teacher phase, the class average and the student with the best knowledge level (the student with the best objective function) are determined. Each student is allowed to communicate with the teacher using the eq. (16).

$$X^{new,i} = X^i + r(X_{teacher} - T_f X_{mean}) \tag{16}$$

The parameter "r" expressed in eq. (16) is a row vector consisting of random numbers ranging from 0 to 1, and its length is equal to the number of design variables. "T_f" is the teaching coefficient and is considered as one of the randomly selected values of 1 or 2. At the end of the teacher phase, it is checked whether the obtained $\psi(x^{new,i})$ value of i.th student is a more appropriate solution than the $\psi(x^i)$ value. If the new calculated value is more

suitable than the old one, $\psi(x^{new,i})$ replaces $\psi(x^i)$. Similarly, the solution vector $X^{new,i}$ replaces the solution vector X^i.

In the student phase, each student in the population must exchange information with other students at least once (Rao et al., 2011). In this case, the i.th student selected from the population interacts with the k.th student (i≠k). Using eq. (17), each student is enabled to interact with other students.

$$\begin{aligned} \psi(X^i) < \psi(X^k) &\Rightarrow X^{new,i} = X^i + r(X^i - X^k) \\ \psi(X^i) > \psi(X^k) &\Rightarrow X^{new,i} = X^i - r(X^i - X^k) \end{aligned} \tag{17}$$

Similarly, if the new calculated value is more suitable than the old one, $\psi(x^{new,i})$ replaces $\psi(x^i)$. Similarly, the solution vector $X^{new,i}$ replaces the solution vector X^i.

Deb's heuristic constraint handling method (Deb, 2000) is used to compare the objective functions calculated at the teacher and student stages with the old objective function (Rao et al., 2011). This comparison takes into account the following 3 rules:

- If a solution (objective function) is a feasible solution (penalty coefficient C=0) and the other solution is an infeasible solution (penalty coefficient C>0), the feasible solution is preferred.
- If both solutions are feasible (penalty coefficient C=0), the solution with the most feasible objective function value is preferred.
- If neither solution is a feasible solution (penalty coefficient C>0), the solution with the lowest penalty coefficient is preferred.

At the end of the accepted iteration (T_{max}), the student who gives the most appropriate objective function is accepted as the solution set and the process is terminated.

3.2. Harmony search algorithm

HSA can be defined as an optimization method obtained by developing a better harmony finding method in musical performance (Geem et al., 2001). Musical harmony is an aesthetic combination of sounds coming out of instruments (design variable) (Geem et al., 2001). In HSA method, each of the solution sets is called a harmony. In eq. (18), i.th harmony (X^i) and design variables are seen.

$$\begin{aligned} X^i &= \left[x_1^i \;\; x_2^i \;\; x_3^i \;\; \ldots x_j^i \right] \\ i &\in (1,2,3,\ldots HMS) \\ j &\in (1,2,3,\ldots D) \end{aligned} \tag{18}$$

In order to make melodies aesthetic, each musical instrument tries to find the most aesthetic melody by producing different sounds with different notes. The notes in the melody are either taken from the note pool or the sounds in the existing harmonies are used. The sounds in the existing harmonies are subjected to pitch if necessary (Lee & Geem, 2004). Firstly, a certain number of harmonies come together to create the harmony memory (HM) (Eq. (19)) and the aesthetic performance of each melody is calculated (objective function).

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_D^1 \\ x_1^2 & x_2^2 & \cdots & x_D^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_D^{HMS} \end{bmatrix} \tag{19}$$

In the second stage, a new harmony is created (Eq. (20)).

$$X^{new} = \left[x_1^{new}, x_2^{new}, \ldots, x_D^{new} \right] \tag{20}$$

The calculation of the design variable of the new harmony depends on three different possibilities. In the first possibility, a random "rand-1" value between 0 and 1 is selected. The condition that the "rand-1" value is greater or less than the Harmony Memory Consideration Ratio (HMCR) is examined (Eq. (21)). If "rand-1" is greater than HMRC, the design variable is selected randomly from the solution space. In the second possibility, if the "rand-1" value is less than or equal to HMRC, the design variable is selected randomly from the melodies in the relevant column in the current HM.

$$x_j^{new} \leftarrow \begin{cases} x_j^{new} \in \left\{ x_j^1, x_j^2, \ldots x_j^{HMS} \right\} \Rightarrow \text{randl} \le \text{HMCR} \\ x_j^{new} \in X \Rightarrow \text{randl} \ge \text{HMCR} \end{cases} \tag{21}$$

In the third possibility, the "rand-2" value randomly selected between 0 and 1 is compared with the pitch adjustment ratio (PAR). If "rand-2" is greater than the PAR value, no change is made to the X_j^{new} value. Otherwise, the pitch adjustment is made to the newly selected design variable as specified in eq. (22). It is rounded to the nearest notes. The amount of rounding is measured by the bandwidth coefficient "b_w".

$$x_j^{new} \leftarrow \begin{cases} = x_j^{new} + b_w * u(-1,1) \Rightarrow \text{random} \le \text{HMCR} \\ = x_j^{new} \Rightarrow \text{random} \ge \text{PAR} \end{cases} \quad (22)$$

The value of u(-1,1) shown in eq (22) represents the random number chosen between -1 and 1. The larger the selected bandwidth, the more the notes ahead or behind are rounded. The aesthetic performance of the newly formed melody is calculated $\psi(x^{new})$ and replaced with the melody with the worst performance in the HM. The calculation step mentioned above is repeated for a predetermined number of iterations.

4. Benchmark studies

In cross-section optimizations using metaheuristic methods, cross-section checks are performed for structural elements after each structural analysis. In addition, if necessary, displacement constraints at nodal points are checked. In such optimizations, while the structural geometry remains constant, element cross-sections can change at each stage. In case of using the finite element Displacement Method (DM), local and global stiffness matrices of the bar elements are calculated and the global stiffness matrix of the system is created at each structural analysis stage. In case of using the Integrated Force Method (IFM), the only expression that needs to be recalculated is the concatenated flexibility matrix seen in eq. (8). The "B" and "C" matrices depend on the structural system geometry and are independent of the element cross-section. For this reason, it is not necessary to calculate them repeatedly at each analysis stage. In order to compare the performance of both analysis methods, examples obtained from the literature containing metaheuristic methods were used. The selected examples belong to plane truss systems. HSA, TLBO, DM and IFM codes are prepared in Python programming language using object-oriented programming (OOP).

4.1. 10 bar plane truss

The plane truss system with 6 nodes and 10 bar elements defined in the X-Z plane is shown in Fig. 3. The elasticity modulus of the material is 10000 ksi and the density of the material is 0.1 lb/in^3. The allowable stress limit for all elements is ±25 ksi, and the displacement limit for the nodes is considered to be ±2 in. Each of the elements will be sized independently. There are a total of 10 design dariables. A force of 100 kips acts on the nodes 2 and 4 in the direction seen in the figure. The structure will be optimized for both continuous and discrete variable sections, and the sections to be used for the discrete value design are shown in Table 1. Continued

10	0.100	0.100	0.100	0.100	0.100	0.100	0.100
Weight (lb)	5057.88	5061.42	5060.97	5064.24	5079.44	5061.13	5061.26
Iteration	20000	-	-	50000.000	50000.000	200	200
Population	-	-	-	-	-	70	70
Harmony	20	20	-	25	25	-	-
HMCR	0.8	0.9	-	0.95	0.95	-	-
PAR	0.3	~	-	0.5	0.5	-	-
bw	-	~	-	0.1	0.1	-	-
CPU (sec.)	-	-	-	157.0625	31.4125	85.6406	23.5625

Table 2.

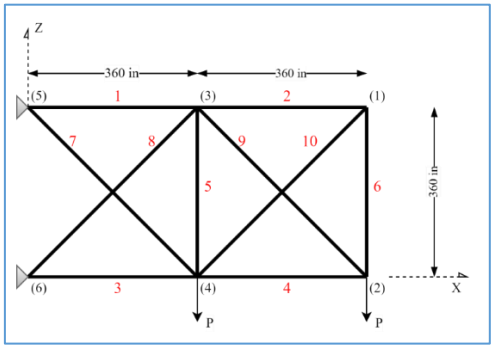


Fig. 3. 10 bar plane truss model

Table 1. Optimization results that containing continuous variables for a 10 bar element truss model

Design variables (in^2)	Lee & Geem (2004)	Değertekin (2012)	Farshchin & Camp (2014)	Current Study (DM)	Current Study (IFM)	Current Study (DM)	Current Study (IFM)
Element	HSA	SAHS	TLBO	HSA	HSA	TLBO	TLBO
1	30.150	30.394	30.668	30.377	30.333	30.725	30.509
2	0.102	0.100	0.100	0.100	0.100	0.100	0.100
3	22.710	23.098	23.158	24.327	23.805	23.253	23.479
4	15.270	15.491	15.223	14.724	14.739	15.260	15.227
5	0.102	0.100	0.100	0.100	0.100	0.100	0.100
6	0.544	0.529	0.542	0.628	0.100	0.571	0.574
7	7.541	7.488	7.465	7.324	8.586	7.443	7.437
8	21.560	21.189	21.026	20.653	20.491	20.854	20.941
9	21.450	21.342	21.466	21.718	21.679	21.510	21.446
Table 1. Continued							
10	0.100	0.100	0.100	0.100	0.100	0.100	0.100
Weight (lb)	5057.88	5061.42	5060.97	5064.24	5079.44	5061.13	5061.26
Iteration	20000	-	-	50000.000	50000.000	200	200
Population	-	-	-	-	-	70	70
Harmony	20	20	-	25	25	-	-
HMCR	0.8	0.9	-	0.95	0.95	-	-
PAR	0.3	~	-	0.5	0.5	-	-
bw	-	~	-	0.1	0.1	-	-
CPU (sec.)	-	-	-	157.0625	31.4125	85.6406	23.5625

Table 2. Discrete variable cross section areas for 10 bar truss

NO	in^2	NO	in^2	NO	in^2
1	1.62	15	3.63	29	11.50
2	1.80	16	3.84	30	13.50
3	1.99	17	3.87	31	13.90
4	2.13	18	3.88	32	14.20
5	2.38	19	4.18	33	15.50
6	2.62	20	4.22	34	16.00
7	2.63	21	4.49	35	16.90
8	2.88	22	4.59	36	18.80
9	2.93	23	4.80	37	19.90
10	3.09	24	4.97	38	22.00
11	3.13	25	5.12	39	22.90
12	3.38	26	5.74	40	26.50
13	3.47	27	7.22	41	30.00
14	3.55	28	7.97	42	33.50

Table 3. Optimization results that containing discreate variables for a 10 bar element truss model

Design Variable (in^2)	Li et al. (2009)	Farshchin & Camp (2014)	Dede (2014)	Current Study (DM)	Current Study (IFM)	Current Study (DM)	Current Study (IFM)
Element	HPSO	TLBO	TLBO	HSA	HSA	TLBO	TLBO
1	30.00	33.50	33.50	33.50	33.50	33.50	33.50
2	1.62	1.62	1.62	1.62	1.62	1.62	1.62
3	22.90	22.90	22.90	22.90	22.90	22.90	22.90
4	13.50	14.20	14.20	14.20	14.20	14.20	14.20
5	1.62	1.62	1.62	1.62	1.62	1.62	1.62

6	1.62	1.62	1.62	1.62	1.62	1.62	1.62
7	7.97	7.97	7.97	7.97	7.97	7.97	7.97
8	26.50	22.90	22.90	22.90	22.90	22.90	22.90
9	22.00	22.00	22.00	22.00	22.00	22.00	22.00
10	1.80	1.62	1.62	1.62	1.62	1.62	1.62
Weight (lb)	5531.98	5490.74	5490.74	5490.74	5490.74	5490.74	5490.74
Iteration	1000	-	50	50000	50000	200	200
Population	50	-	20	-	-	70	70
Harmony	20	20	-	25	25	-	-
HMCR	0.8	0.9	-	0.95	0.95	-	-
PAR	0.3	~	-	0.5	0.5	-	-
CPU (sec.)	-	-	-	156.3281	47.5625	85.0938	26.9063

4.2. 15 bar plane truss

The plane truss system with 8 nodes consisting of 15 bar elements defined in the X-Z plane is shown in Fig. 4. The elasticity modulus of the material is 200000 MPa and the density of the material is 7860 kg/m^3. The allowable stress limit for all elements is ±120 MPa and the displacement limit for the nodes is considered to be ±10 mm. Each of the elements will be sized independently and there are 15 design variables in total. A force of 35 kN acts on the nodes 4, 6 and 8 in the direction seen in the figure. The truss system will be optimized for discrete variable sections. The sections to be used in the design are shown in Table 4.

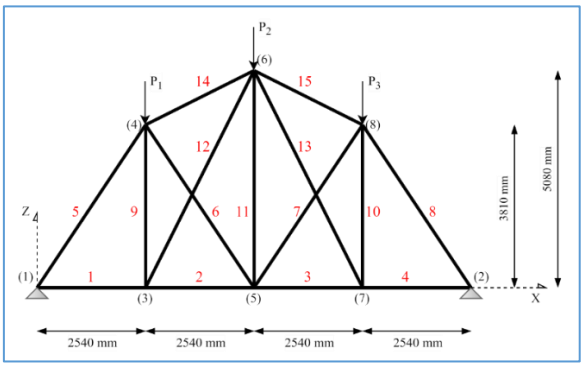


Fig. 4. 15 bar plane truss model

Table 4. Discrete variable cross section areas for 15 bar truss

NO	mm^2	NO	mm^2
1	113.2	9	308.6
2	143.2	10	334.3
3	145.9	11	338.2
4	174.9	12	497.8
5	185.9	13	507.6
6	235.9	14	736.7
7	265.9	15	791.2
8	297.1	16	1063.7

Table 5. Optimization results that containing discreate variables for a 15 bar plane truss system

Design Variable (in^2)	Dede (2014)	Current Study (DM)	Current Study (BKM)	Current Study (DM)	Current Study (BKM)
Element	TLBO	HSA	HSA	TLBO	TLBO
1	113.20	113.20	113.20	113.20	113.20
2	113.20	143.20	113.20	113.20	113.20
3	113.20	113.20	113.20	113.20	113.20

4	113.20	113.20	113.20	113.20	113.20
5	736.70	736.70	736.70	736.70	736.70
6	113.20	113.20	113.20	113.20	113.20
7	113.20	113.20	113.20	113.20	113.20
8	736.70	736.70	736.70	736.70	736.70
9	113.20	113.20	113.20	113.20	113.20
10	113.20	113.20	113.20	113.20	113.20
11	113.20	113.20	113.20	113.20	113.20
12	113.20	113.20	113.20	113.20	113.20
13	113.20	113.20	113.20	113.20	113.20
14	334.30	338.20	338.20	334.30	334.30
15	334.30	338.20	334.30	334.30	334.30
Weight (kg)	105.735	106.50	105.82	105.735	105.735
Iteration	50	50000	50000	50	50

Tablo 5. Continued

Population	30	-	-	30	30
Harmony	-	30	30	-	-
HMCR	-	0.95	0.95	-	-
PAR	-	0.5	0.5	-	-
CPU (sec.)	-	253.0625	61.5938	13.8438	2.9375

Table 6. Discrete variable cross section areas for 52 bar truss

NO	mm^2	NO	mm^2	NO	mm^2	NO	mm^2
1	71.613	17	1008.385	33	2477.414	49	7419.34
2	90.968	18	1045.159	34	2496.769	50	8709.66
3	126.451	19	1161.288	35	2503.221	51	8967.72
4	161.290	20	1283.868	36	2696.769	52	9161.27
5	198.064	21	1374.191	37	2722.575	53	9999.98
6	252.258	22	1535.481	38	2896.768	54	10322.56
7	285.161	23	1690.319	39	2961.284	55	10903.20
8	363.225	24	1696.771	40	3096.768	56	12129.01
9	388.386	25	1858.061	41	3206.445	57	12838.68
10	494.193	26	1890.319	42	3303.219	58	14193.52
11	506.451	27	1993.544	43	3703.218	59	14774.16
12	641.289	28	729.031	44	4658.055	60	15806.42
13	645.160	29	2180.641	45	5141.925	61	17096.74
14	792.256	30	2238.705	46	5503.215	62	18064.48
15	816.773	31	2290.318	47	5999.988	63	19354.80
16	939.998	32	2341.931	48	6999.986	64	21612.86

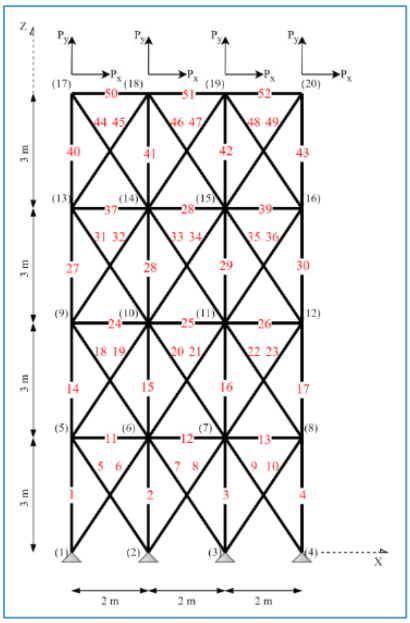


Fig. 5. 52 bar plane truss model

4.3.52 bar plane truss

The plane truss system with 20 nodes consisting of 52 bar elements defined in the X-Z plane is shown in Fig. 5. The elasticity modulus of the material is 207000 MPa and the material density is 7860 kg/m^3. The allowable stress limit for all elements is ±180 MPa, and there is no displacement limit for nodes. It will be sized in groups and there are 12 design variables in total. The nodes numbered 17, 18, 19 and 20 are subject to a force of 100 kN in the X direction and 200 kN in the Z direction. The carrier system will be optimized for discrete variable sections. The sections to be used for the discrete value design are shown in Table 6.

Table 7. Optimization results that containing discreate variables for a 52 bar element truss model

Design Variable (in^2)		Lee et al. (2005)	Dede (2014)	Current Study (DM)	Current Study (BKM)	Current Study (DM)	Current Study (BKM)
Group	Element	HS	TLBO	HSA	HSA	TLBO	TLBO
1	1-4	4658.055	4658.055	4658.055	4658.055	4658.055	4658.055
2	5-10	1161.288	1161.288	1161.288	1161.288	1161.288	1161.288
3	11-13	506.451	494.193	388.386	388.386	494.193	494.193
4	14-17	3303.219	3303.219	3303.219	3303.219	3303.219	3303.219
5	18-23	939.998	939.998	939.998	939.998	939.998	939.998
6	24-26	494.193	494.193	729.031	506.451	494.193	494.193
7	27-30	2290.318	2238.705	2238.705	2238.705	2238.705	2238.705
8	31-36	1008.385	1008.385	1008.385	1008.385	1008.385	1008.385
9	37-39	2290.318	494.193	388.386	363.225	494.193	494.193
10	40-43	1535.481	1283.868	1283.868	1283.868	1283.868	1283.868
11	44-49	1045.159	1161.285	1161.288	1161.288	1161.285	1161.285
12	50-52	506.451	494.193	494.193	729.031	494.193	494.193
Weight (kg)		1906.760	1902.605	1903.710	1903.102	1902.605	1902.605
Iteration		30000	100	50000	50000	120	120
Population		-	60	-	-	60	60
Harmony		20	-	30	30	-	-
HMCR		0.9	-	0.95	0.95	-	-
PAR		0.45	-	0.5	0.5	-	-
CPU (sec.)		-	-	775.8125	140.125	216.0625	36.0938

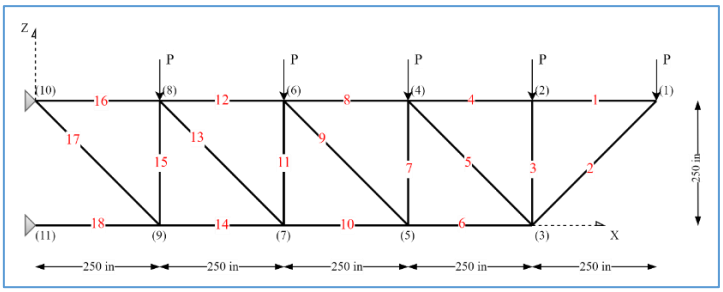


Fig. 6. 18 bar plane truss model

Table 8. Optimization results that containing continuous variables for a 18 bar element truss model

Design Variable (in^2)		Lee & Geem (2004)	Current Study (DM)	Current Study (BKM)	Current Study (DM)	Current Study (BKM)
Group	Element	HS	HSA	HSA	TLBO	TLBO
1	1, 4, 8, 12, 16	9.980	10.000	10.000	10.000	10.000
2	2, 6, 10, 14, 18	21.630	21.651	21.651	21.651	21.651
3	3, 7, 11, 15	12.490	12.500	12.500	12.500	12.500
4	5, 9, 13, 17	7.057	7.072	7.072	7.072	7.072
Weight (kg)		6421.880	6430.969	6430.969	6430.913	6430.521
Iteration		2000	20000	20000	100	100
Population		-	-	-	20	20
Harmony		20	20	20	-	-
HMCR		0.8	0.95	0.95	-	-
PAR		0.3	0.3	0.3	-	-

Tablo 8. Continued

bw	-	0.1	0.1	-	-
CPU (sec.)	-	105.5469	17.4844	20.4375	3.1406

4.4.18 bar plane truss

The plane truss system with 11 nodes consisting of 18 rod elements defined in the X-Z plane is shown in Fig. 6. The elasticity modulus of the material is 10000 ksi, and the material density is 0.1 lb/in^3. The allowable stress limit for all elements is ±20 ksi, and there is no displacement limit for the nodes. In addition to the given stress limit, Euler buckling stress $\left(\sigma_b^i = -4EA_i/L_i^2\right)$ will be taken into account for rod elements under compression. The system will be dimensioned in groups and there are 4 design variables in total. A force of -20 kips acts in the Z direction as seen in the figure from the nodes 1, 2, 4, 6 and 8. The system will be optimized for continuously variable cross-sections.

5. Results and discussion

Under the headings 4.1, 4.2, 4.3 and 4.4, plane truss models were examined by considering continuous and discrete design variables. When the obtained results were compared with the benchmark studies, it was concluded that the prepared structural analysis and optimization codes provided sufficiently accurate data.

In terms of optimization data input parameters, the required user data input parameters for the TLBO method are the population number and the iteration number. The required parameters in the HSA method are the harmony number, iteration number, HMCR and PAR values. The TLBO method offers simplicity in terms of user data input. In the HSA method, it should be taken into account that the HMCR and PAR parameters may vary depending on the structural model. Since there is no single acceptance for these values, it is necessary to check and verify them over different values even in the same structural model.

When the data obtained in terms of analysis times are compared, it is determined that structural analysis performed with IFM are 6-8 times faster than DM. The reason for this circumstances is that, as mentioned before, in fixed geometry section optimizations, when IFM is used, the only expression that changes is the concatenated flexibility matrix. It should be noted that the analysis times obtained in this study may vary from computer to

computer depending on the hardware performance. Even in such a case, it will not change the fact that the CPU time rates of both analysis methods are at the same levels.

6. Conclusion

In this study, IFM and DM were compared in terms of CPU time under iterative structural analysis. Since the most intensive use of iterative analysis is metaheuristic optimization methods, two metaheuristic optimization methods were selected and cross section optimizations were performed on plane truss models. The results obtained were compared with the results in the literature. As a result of the comparisons made, it was concluded that the expected goals were achieved and IFM was more efficient in terms of CPU times compared to DM in iterative processes.

It should be noted that the obtained results are valid for fixed geometry plane truss systems. Because, it should not be forgotten that in case of geometry optimizations, the equilibrium and compatibility matrices must be recalculated at each stage.

References

Artar, M. (2015). *Optimization of steel frames with composite beams using genetic algorithm* [Doctoral dissertation, Karadeniz Technical University].

Artar, M., & Daloğlu, A. T. (2018). Optimum weight design of steel space frames with semi-rigid connections using harmony search and genetic algorithms. *Neural Computing and Applications, 29*(11), 1089–1100.

Camp, C. V., & Farshchin, M. (2014). Design of space trusses using modified teaching–learning-based optimization. *Engineering Structures, 62–63*, 87–97.

Deb, K. (2000). An efficient constraint handling method for genetic algorithm. *Computer Methods in Applied Mechanics and Engineering, 186*, 311–338.

Degertekin, S. O. (2012). Improved harmony search algorithms for sizing optimization of truss structures. *Computers and Structures, 92–93*, 229–241.

Dede, T. (2014). Application of teaching-learning-based-optimization algorithm for the discrete optimization of truss structures. *KSCE Journal of Civil Engineering, 18*(6), 1759–1767.

Geem, Z. W., Kim, J. H., & Loganathan, G. V. (2001). A new heuristic optimization algorithm: Harmony search. *Simulation, 76*(2), 60–68.

Hasançebi, O., & Saka, M. P. (2010). Optimum design of three dimensional steel structural systems using stochastic optimization methods and its application to structural engineering in our country. Retrieved from https://open.metu.edu.tr/bitstream/handle/11511/50065/TVRBNU9UTXo.pdf

Lee, K. S., & Geem, Z. O. (2004). A new structural optimization method based on the harmony search algorithm. *Computers and Structures, 82*, 781–798.

Lee, K. S., & Geem, Z. O. (2005). A new meta-heuristic algorithm for continuous engineering optimization: Harmony search theory and practice. *Computer Methods in Applied Mechanics and Engineering, 194*, 3902–3933.

Lee, K. S., Geem, Z. O., Lee, S. H., & Bae, K. W. (2005). The harmony search heuristic algorithm for discrete structural optimization. *Engineering Optimization, 37*(7), 663–684.

Li, L. J., Huang, Z. B., & Liu, F. A. (2009). A meta-heuristic particle swarm optimization method for truss structures with discrete variables. *Computers and Structures, 87*, 435–443.

Patnaik, N. S. (1973). An integrated force method for discrete analysis. *International Journal for Numerical Methods in Engineering, 6*, 237–251.

Rao, R. A., Savsani, V. J., & Vakharia, D. P. (2011). Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems. *Computer-Aided Design, 43*, 303–315.

Topçu, A. (2017). Element types and matrices part 22. Retrieved from https://atopcu.osmanmuratkaya.com/index_dosyalar/Dersler/SonluElemanlarMetodu/SEM22_KUVVET%20METODU_ElemanMatrisleri.pdf