ON COMPLEMENT OF BASE IN B_3

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ABSTRACT

This article is about the base complement in B_3. The complement method consists of the course of operations around the centroid. The centroid is played an important role in the encoding of numbers. Some of the numbers are smaller than the centroid and some are larger than the centroid. Thus subtraction of any number is implemented by adding its complement. The concept of caryy comes into play when numbers smaller than the centroid are subtracted from numbers larger than the centroid. The pairs of mutually additive inverse numbers are called complements. Changing the sign of any number is encoded by generating its complement, which can be with a very simple and useful known algorithm. This method is widely used in digital computing.

The base complement of an $n-digit$ number y in base r is defined as $r^n - y$. In practice, the base complement is $(r^n - 1) - y$.

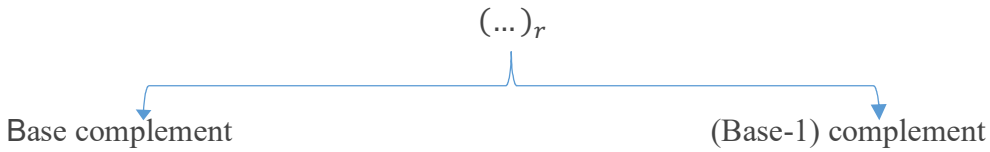
All these calculations were made in $\boldsymbol{B_2}$. The study is about the role of the centroid concept in $\boldsymbol{B_3}$. The digit value of a number in the ternary system is redefined. The new definition radix and diminished radix complement is also updated for this definition. This situation enriched the coding. Any number is written $\mathbf{0}$ and $\mathbf{1}$ in $\boldsymbol{B_2}$. This number is expressed $\mathbf{-1}$, $\mathbf{0}$ and $\mathbf{1}$ in $\boldsymbol{B_3}$. The expression of a number as $\mathbf{-1}, \mathbf{0}$ and $\mathbf{1}$ offers new gains in the coding and processing of data. The preliminary approaches in the study are presented noteworthy cases in the evaluation of approaches in numeric and alphanumeric expressions. Initial statements are given on innovations in coding and operations.

Keywords : base, complement, radix, , diminished, coding.

1. INTRODUCTION

In B_2, there are two main types of complements for any number system and they are

i. Base complement
ii. (Base-1) complement



First method of base complement operation addition is given number proceeds as follow:

If there are any zeros from right to left of the number, the first non-zero digit after the zeros is subtracted from the base r. The other digits are subtracted from base-1 respectively.

Example: If number is $-abcd0$ and base is r, then

$$r - d = e$$
$$(r-1) - c = f$$
$$(r-1) - b = g$$
$$(r-1) - a = h$$

The complement of base r number $-abcd0$ is $hgfe0$.

Second method of base $(\boldsymbol{r-1})$ complement is given number proceeds as follow:

If r is the base, then $(base - 1)$ value is $(r-1)$. Therefore $(r-1)'s$ complement is:

$$(r-1)(r-1)(r-1)(r-1) - \ abcd0 = \cdots$$

Let $a > 1$ be an integer and $c_0, c_1, \dots, c_n \in \{0, \dots, a-1\}$ for $n \in \mathbb{Z}^+$. Any $k \in \mathbb{Z}$ is written uniquely in the form

$$k = \sum_{i=1}^{n} c_i a^i, \text{ where } n \in \mathbb{Z}^+.$$

Here a is the base and $c_0, c_1, \dots, c_n$ are the digits.

Definition 1.1. The integer k is called a centroid with coefficient b in base a if $k = ba^s$ for constant integers a, b, where $s \in \mathbb{Z}^+$.

Here the integer a is called the base of the centroid and the integer s is called the period $p(a)$ of the centroid. The period of elements with the same number of digits is considered as the positive integer s. The set of these elements is called the centroid set and is denoted by $C_{B_a}(k)$. The number of elements in $C_{B_a}(k)$ is denoted by $|C_{B_a}(k)|$. The number of coefficients of integer k written in base a is denoted by $d_{B_a}(k)$.With this notation, we have

$$C_{B_a}(k) = \{c \in \mathbb{Z}^+ | d_{B_a}(k) = d_{B_a}(c)\}.$$

In our study, $a = 3$ and $b = 1$ are taken since the ternary number system is used in [3].

The following theorem is given without proof in [3].

Theorem 1.2. Let $a > 1$ be an integer and $c_0, c_1, \ldots, c_n \in \{-1, 0, 1\}$ for $n \in \mathbb{Z}^+$. Any $k \in \mathbb{Z}$ is written uniquely in the form

$$k = \sum_{i=0}^{n} c_i a^i, \text{ where } n \in \mathbb{Z}^+.$$

If any number is written using this theorem, this number is called to be written in base B_3. It is denoted by

$$k = (\ldots)_{B_3}.$$

The values and complements of some numbers are given in base B_2 and base B_3 at in the table below.

Number	B_2	Complement of B_2	B_3
0	00	1010	00
1	01	1001	01
2	10	1000	1-1
3	11	111	10
4	100	110	11
5	101	101	1-1-1
6	110	110	1-10
7	111	111	1-11
8	1000	10	10-1
9	1001	01	100
10	1010	00	101
11	1011	111001	11-1
12	1100	1011000	110
13	1101	1010111	111
14	1110	1010110	1-1-1-1
15	1111	1010101	1-1-10
16	10000	1010100	1-1-11
17	10001	1000011	1-10-1
18	10010	1000010	1-100
19	10011	1000001	1-101
20	10100	1010000	1-11-1

21	10101	1001111	1-110
22	10110	1001110	1-111
23	10111	1001101	10-1-1
24	11000	1001100	10-10
25	11001	1001011	10-11
26	11010	1001010	100-1
27	11011	1001001	1000
28	11100	1001000	1001
29	11101	1000111	101-1
30	11110	1000110	1010
31	11111	1000101	1011
32	100000	1000100	11-1-1
33	100001	1000011	11-10
34	100010	1000010	11-11
35	100011	1000001	110-1
36	100100	1000000	1100
37	100101	111111	11011
38	100110	111110	111-1
39	100111	111101	1110
40	101000	111100	1111
41	101001	111011	1-1-1-1-1

Proposition 1.3. Let $x \in C_{B_a}(k)$be a centroid. If $x \in C_{B_a}(k)$ then

$$c_0, c_1, \ldots, c_n \in \{0, 1\}.$$

Proof. If $x \in C_{B_a}(k)$ then

$$x = \sum_{i=0}^{n} c_i a^i, \text{ where } n \in \mathbb{Z}^+.$$

$$x = (c_n \ldots c_1 c_0)_r,$$

For $0 \leq i \leq n-1$, $c_i = 0$ and $c_n = 1$. Because,

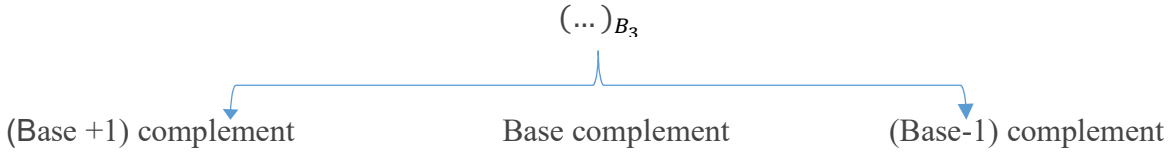
$$x = a^n.$$

Proposition 1.4. Let $C_{B_a}(k)$ be a centroid and $x \in C_{B_a}(k)$. If $x = a^s$ then The number of digits of x is s.

2. COMPLEMENT OF BASE IN B_3

In B_3, the complements of a number are $base + 1$ complement, $base$ complement and $base - 1$ complement.

The definition of $base + 1$ a number is $a^s + 1$. The $base + 1$ complement of x is $a^s + 1 - x = y$. The $base$ complement of x is $a^s - x = y$. And The $base - 1$ complement of x is $a^s - 1 - x = y$, where s is the number of digits.



Number	B_3	Complement of B_3		
		(Base +1) complement	Base complement	(Base-1) complement
0	00	110	101	100
1	01	101	100	10-1
2	1-1	100	10-1	1-11
3	10	10-1	1-11	1-10
4	11	1-11	1-10	1-1-1
5	1-1-1	1-10	1-1-1	011
6	1-10	1-1-1	011	010
7	1-11	011	010	01-1
8	10-1	010	01-1	001
9	100	01-1	001	000
10	101	10101	10100	1010-1
11	11-1	10100	1010-1	101-11
12	110	1010-1	101-11	101-10
13	111	101-11	101-10	101-1-1
14	1-1-1-1	101-10	101-1-1	10011
15	1-1-10	101-1-1	10011	10010
16	1-1-11	10011	10010	1001-1
17	1-10-1	10010	1001-1	10001

18	1-100	1001-1	10001	10000
19	1-101	10001	10000	1000-1
20	1-11-1	10000	1000-1	100-11

These digit complements, respectively, follow one another like the rungs of a staircase.

Lemma 2.1. Between the digit complements of any integer, the following holds.

If the $base\ +1$ complement of x is $a^s + 1 - x = y$, then

i. The $base$ complement of x is $a^s - x = y - 1$.
ii. The $base - 1$ complement of x is $a^s - 1 - x = y - 2$.

Proof. The proof of the lemma is clear.

3. RESULTS AND CONCLUSIONS

Logic B_3 offers new approaches to the coding system. Some of the rules logic are still open problems.

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