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Structural parameter identification of fixed end beams by inverse method using measured natural frequencies

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Abstract. Structural parameter identification based on the measured dynamic responses has become very popular recently. This paper presents structural parameter identification of fixed end beams by inverse method using measured natural frequencies. An added mass is used as a modification tool. The measurements of the flexural vibrations of a fixed end beam with and without added mass are performed by using experimental modal testing. The solution of free bending transverse vibration of the beam is obtained by solving the differential equation motion of Bernoulli-Euler beam. By introducing the natural frequencies from experimental measurements into the solution of differential equation, the structural parameters of the fixed end beam are calculated.

It is seen from the results that the values of the mass distribution and elasticity modulus identified using the first natural frequency of the beam nearly close to the real values. Besides, the theoretical frequencies obtained using the identified structural parameters also close to the measured frequencies.

Keywords: Added mass, beam, experimental modal analysis, flexural vibration, inverse method, structural parameter identification

1. Introduction

Structural behaviour is affected by the structural material, boundary conditions, and loading type. For this reason, the true determination of the structural response is highly difficult. Theoretical and experimental methods are commonly used for the determination of structural behaviour. The most effective method is the interactive using of the theoretical and experimental methods. Thus, the more reliable result can be obtained. Bagchi [1], Cunha and Piranda [2], and Steenackers and Guillaume [3] used the experimental modal properties to improve analytical models in better prediction structural behaviour.

The responses of a beam can be obtained by solving the Bernoulli-Euler beam equation. The parameters contained in this equation such as elasticity modulus, mass distribution, and inertia moment of the cross-section are determined depending on the dimension and material used in the structure. However, in most cases they are assumed to have standard value, for example $E = 2 \times 10^{11}$Pa for steel. Then the responses of the structure are determined depending on the standard value. When taken into account some special structures, such as bridges, the true determination of structural parameters become important because of the fact that the structural response is determined depending on the structural parameters. The inverse method is commonly used in the determination of structural parameters. In this method, the natural frequencies obtained from the experimental measurement are introduced into the equation, which is the differential solution of Bernoulli-Euler equation, and then structural parameters are identified. The inverse method bases on the measurement of natural frequencies with different cases; original and modified structure.

The determination of structural parameter has been studied using analytical, numerical, and experimental methods. The identification of structural material properties by using the measured modal results was presented by Sol et

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al. [4]. Lauwagie et al. [5] proposed a method to estimate the uncertainty associated with the elastic properties identified with a Mixed-Numerical-Experimental-Technique (MNET). Elasticity and transverse shear modulus of beams were obtained from the measured flexural resonance frequencies using inverse method by Shi et al. [6] and Larsson [7]. Lauwagie et al. [8] presented a procedure to damage identification in beams using inverse method.

Kubojima et al. [9] investigated the influence of a concentrated mass attached to a wooden beam on its Young's modulus based on the Bernoulli-Euler beam elementary theory of bending. Gladwell [10] investigated the effect of boundary conditions on the response spectra and obtained mass and stiffness matrix coefficient for cantilever beam. Skrinar and Umek [11] used masses instead of boundary change as a modification tool to provide the controlled change. The method was applied on a simple supported beam to identify the structural parameters. Encouraged by the experimental results, the theoretical extension of the previously presented method requiring a smaller number of input data, particularly suitable for cantilevers, was introduced later by Skrinar and Umek [12]. G ürgöze et al. [13] investigated the bending eigenfrequencies of a cantilever Bernoulli-Euler beam with a tip mass. The exact and approximate frequency equations were derived from the boundary value formulation and the Lagrange's multipliers method, respectively. Beams carrying a concentrated mass were investigated by Low [14] and Chai and Low [15]. Bayraktar et al. [16] also investigated the effect of added mass on the response of structure by experimental modal analysis method, not in the sense of inverse identification, but from the eigenfrequency determination point of view. A fixed end beam was taken into account in the applications. Experimental results were verified with the theoretical results. Skrinar and Umek [17] presented two procedures for the prediction of changes in eigenfrequencies caused by a mass added on simply supported beams. Skrinar [18] also presented the inverse parameter identification of a simply supported beam. In this study, the structural parameters were obtained within the engineering accuracy. Kubojima et al. [19] obtained the resonance frequencies in timber guardrail beams between the simply supported condition and fixed condition from flexural vibration tests.

In this paper, it is presented a study of the structural parameter identification of a fixed end beam with uniform distributed mass and flexural rigidity EI by inverse method. A mathematical description of the system is the essential part of the method. The mathematical solution of the beam is derived from the solution of Bernoulli-Euler beam equation. To determine the structural parameter, the natural frequencies of original and modified structure must be measured. The experimental modal analysis method is used to determine the natural frequencies of the beam. The modified structure is established by using an added mass, which have rectangular shape and is made of steel. The mass and mass moment of inertia of the added body are determined and then introduced into the differential equation of motion to identify the modified structural response.

2. Formulations

2.1. Differential formulation of original and modified beam

In this study, a fixed end beam is selected to apply the inverse parameter identification method. These types of beams are used in many engineering structures. The eigenfrequencies of the beam with uniform material properties can be obtained through the closed mathematical solution. Considering an elementary case that neglects shear, damping, and axial-force effects, the solution of free bending transverse vibration of a uniform homogeneous beam is obtained by solving the differential equation of motion of a Bernoulli-Euler beam that can be written as follows [20],

$$\mathrm{EI}\frac{\partial^4 \nu}{\partial x^4} + \mathrm{m}\frac{\partial^2 \nu}{\partial t^2} = 0 \tag{1}$$

where E is the elasticity modulus of material, I is the inertia moment of cross section, m is the mass per unit length, and v is the transverse displacement of beam. The solution of Eq. (1) is,

$$v(x,t) = K(x)Y(t) \tag{2}$$

Equation (2) is introduced into Eq. (1) and then Eq. (1) is simplified as,

$$\mathrm{EI}\ K^{(4)}(x)Y(t) + \mathrm{m}\,\ddot{Y}(t)K(x) = 0 \tag{3}$$

and can be written as follows,

$$\frac{\ddot{Y}(t)}{Y(t)} + \frac{\mathrm{EI}}{\mathrm{m}} \frac{K^{(4)}(x)}{K(x)} = 0 \tag{4}$$

The solution of the above equation requires that both sides are equal to a constant, for example ω^2,

$$\frac{\ddot{Y}(t)}{Y(t)} = -\omega^2 \tag{5}$$

$$\frac{\mathrm{EI}}{\mathrm{m}} \frac{K^{(4)}(x)}{K(x)} = \omega^2 \tag{6}$$

The partial differential Eq. (1) is thus transformed into two ordinary differential Eqs (5) and (6) with two unknown functions $K(x)$ and $Y(t)$.

$$K^{(4)}(x) - \omega^2 \frac{\mathrm{m}}{\mathrm{EI}} K(x) = 0 \tag{7}$$

Introducing

$$\lambda^4 = \omega^2 \frac{\mathrm{m}}{\mathrm{EI}} \tag{8}$$

The differential Eq. (7) can be written as,

$$K^{(4)}(x) - \lambda^4 K(x) = 0 \tag{9}$$

The solution of the equation is,

$$K(x) = A_1 \sinh \lambda x + A_2 \cosh \lambda x + A_3 \sin \lambda x + A_4 \cos \lambda x \tag{10}$$

The constants (A_1, A_2, A_3, A_4) in this equation can be determined from boundary conditions. These boundary conditions for a fixed end beam are,

$$\left.\begin{array}{l} K(x) = 0 \\ \frac{dK}{dx} = 0 \end{array}\right\} \text{ at } x = 0 \qquad \left.\begin{array}{l} K(x) = 0 \\ \frac{dK}{dx} = 0 \end{array}\right\} \text{ at } x = \mathrm{L} \tag{11}$$

where L is the length of the beam.

The four boundary conditions are introduced into Eq. (10) to determine the unknown constant. To determine the eigenfrequencies, the problem generally is reduced into the solutions of four homogenous linear equations. The system has a nontrivial solution if the determinant of the characteristic equation of the system is equal to zero. The nth circular frequency of the fixed end beam is obtained in the form of as follow [20],

$$\omega = (\lambda L)_n^2 \sqrt{\frac{1}{\mathrm{L}^4} \frac{\mathrm{EI}}{\mathrm{m}}}, \quad \begin{cases} (\lambda L)_1 = 4.730 \\ (\lambda L)_2 = 7.853 \\ (\lambda L)_3 = 10.996 \\ (\lambda L)_n = \frac{1}{2}(2 * n + 1)\pi, \ n > 3 \end{cases} \tag{12}$$

It is seen from the above equation that the eigenfrequencies depend on the L, EI, and m. The (EI/m) ratio is obtained in the form of,

$$\frac{\mathrm{EI}}{\mathrm{m}} = \left(\frac{\omega \mathrm{L}^2}{(\lambda L)_n^2} \right)^2 \tag{13}$$

If the natural frequencies of the system are measured experimentally and the length of the beam is determined accurately, the EI/m ratio can be calculated easily from the above equation. However, the exact values of E, I, and m remain unknown. To solve the problem, an additional equation is required that can be obtained from a structure with controlled change.

To determine the exact values of E, I, and m, an added mass is used in this study. The added body with the mass m_a and the mass moment of inertia J_a is located at a known distance as shown in Fig. 1. The solving process is

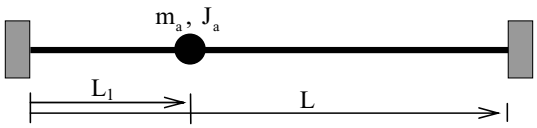


Fig. 1. A fixed end beam with added mass.

similar as in the case of without added mass with the difference that the added mass separates the beam into two parts. Then, the $K(x)$ functions in Eq. (10) consists of the $K_1(x)$ and $K_2(x)$ for the left part and right part of the structure, respectively.

$$K_1(x) = B_1 \sinh \lambda x + B_2 \cosh \lambda x + B_3 \sin \lambda x + B_4 \cos \lambda x \tag{14}$$

$$K_2(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \cos \lambda x \tag{15}$$

where the constant λ was already given by Eq. (8) and ω now represents the circular eigenfrequency of the structure with added mass. To determine the eight coefficients B_i and C_i ($i = 1 - 4$), four boundary conditions and four compatibility equations at the added mass location are taken into account. The eigenfrequencies of the system are obtained from the solution of eight homogenous equations.

The four boundary conditions describing the displacements and rotations at both ends and the four compatibility equations at the added mass location describing the displacements, rotations, bending moments, and shear forces are given below,

$$\left.\begin{array}{l} K_1(0) = 0 \\ \frac{dK_1(0)}{dx} = 0 \\ K_2(\mathrm{L}) = 0 \\ \frac{dK_2(\mathrm{L})}{dx} = 0 \end{array}\right\} \text{ Boundary Conditions} \tag{16}$$

$$\left.\begin{array}{l} K_1(\mathrm{L}_1) = K_2(\mathrm{L}_1) \\ \frac{dK_1(\mathrm{L}_1)}{dx} = \frac{dK_2(\mathrm{L}_1)}{dx} \\ \mathrm{EI}\left[\frac{d^2K_1(\mathrm{L}_1)}{dx^2} - \frac{d^2K_2(\mathrm{L}_1)}{dx^2}\right] - \mathrm{J}_a\omega^2 \frac{dK_1(\mathrm{L}_1)}{dx} = 0 \\ \mathrm{EI}\left[\frac{d^3K_1(\mathrm{L}_1)}{dx^3} - \frac{d^3K_2(L_1)}{dx^3}\right] + m_a\omega^2 K_1(L_1) = 0 \end{array}\right\} \text{ Compatibility Equations} \tag{17}$$

Introducing the boundary conditions related to displacements in Eq. (16) into Eqs (14–15) yields the below expressions,

$$\begin{array}{l} A_1 = -A_3 \\ A_2 = -A_4 \end{array} \tag{18}$$

The remaining boundary conditions and compatibility equations result in a system of homogenous linear equations. Such a system has always a trivial solution with no engineering meaning and a non-trivial solution when the determinant of the system is equal to zero. Therefore, the determinant of the following matrix must vanish,

$$\begin{bmatrix} 0 & 0 & sh_L & ch_L & s_L & c_L \\ 0 & 0 & ch_L & sh_L & c_L & -s_L \\ s_1 - sh_1 & c_1 - ch_1 & -sh_1 & -ch_1 & -s_1 & -c_1 \\ c_1 - ch_1 & -s_1 - sh_1 & -ch_1 & -sh_1 & -c_1 & s_1 \\ -k_1(s_1 + sh_1) - \mathrm{J}_a\omega^2(c_1 - ch_1) & -k_1(c_1 + ch_1) + \mathrm{J}_a\omega^2(s_1 + sh_1) & -k_1 sh_1 & -k_1 ch_1 & k_1 s_1 & k_1 c_1 \\ -k_2(c_1 + ch_1) + \mathrm{m}_a\omega^2(s_1 - sh_1) & k_2(s_1 - sh_1) + \mathrm{m}_a\omega^2(c_1 - ch_1) & -k_2 ch_1 & -k_2 sh_1 & k_2 c_1 & -k_2 s_1 \end{bmatrix} \tag{19}$$

with symbols

s_1=sin($\lambda \mathrm{L}_1$), sh_1=sinh($\lambda \mathrm{L}_1$), s_L=sin(λL), sh_L=sinh(λL)

c_1=cos($\lambda \mathrm{L}_1$), ch_1=cosh($\lambda \mathrm{L}_1$), c_L=cos(λL), ch_L=cosh(λL)

k_1=EIλ, k_2=EIλ^3

From the determinant of the matrix, the EI/m ratios of the beam are obtained in a complicated form. Much simpler expressions can be found for some particular locations. If the added mass is located in the middle of the beam, the governing equation, due to the symmetry of the structure, reduces to a form of two products,

$$\text{EI} = \frac{m_a\omega^2(1-\cos(0.5L\lambda)\cosh(0.5L\lambda))}{\lambda^3(2\cosh(0.5L\lambda)\sin(0.5L\lambda)+2\cos(0.5L\lambda)\sinh(0.5L\lambda)} \tag{20}$$

$$\text{EI} = \frac{J_a\omega^2(-1+\cos(0.5L\lambda)\cosh(0.5L\lambda))}{\lambda(-2\cosh(0.5L\lambda)\sin(0.5L\lambda)+2\cos(0.5L\lambda)\sinh(0.5L\lambda)} \tag{21}$$

The first equation is only the function of mass of added body and belongs to the symmetrical modes and related eigenfrequencies. The second equation is influenced by the moment of inertia of added mass and belongs to the antisymmetric mode shapes and related eigenfrequencies.

2.2. Theoretical and experimental modal analysis of the beam

The natural frequencies are obtained by using the equation of motion. For undamped vibration of a multi degrees of freedom system, the equation of motion in matrix form is expressed as follows [21],

$$[M]\ \{\ddot{v}(t)\} + [K]\ \{v(t)\} = \{F(t)\} \tag{22}$$

where $[M]$ and $[K]$ are mass and stiffness matrices, $\{\ddot{v}(t)\}$ and $\{v(t)\}$,and $\{F(t)\}$ are vectors of time varying acceleration, displacement, and load vectors, respectively. A typical two degrees of freedom system with added mass is given in Fig. 2.

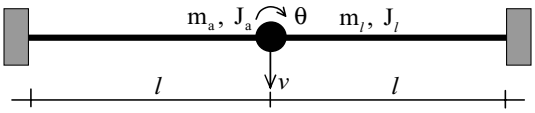


Fig. 2. A typical two degrees of freedom system with added mass.

The mass and stiffness matrixes for this system are defined as follows,

$$[M] = \begin{bmatrix} m_l & 0 \\ 0 & J_l \end{bmatrix} + \begin{bmatrix} m_a & 0 \\ 0 & J_a \end{bmatrix} \tag{23}$$

$$[K] = \begin{bmatrix} 24^{EI}/_{l^3} & 0 \\ 0 & 8^{EI}/_{l} \end{bmatrix} \tag{24}$$

In these equations, m_l and J_l are the lumped mass and moment of inertia of lumped mass; m_a and J_a are the added mass and moment of inertia of added mass, respectively.

If free vibration form of Eq. (22) is premultiplied by the inverse of the mass matrix $[M]^{-1}$, then

$$[I]\{\ddot{v}(t)\} + [D]\{v(t)\} = 0 \tag{25}$$

where $[I]$ is the unit matrix. The dynamic matrix $[D]$ is given by

$$[D] = [M]^{-1}[K] \tag{26}$$

Then, theoretical eigenfrequencies are obtained by using the characteristic equation of the system as follows [21],

$$\left|[D] - \omega^2[I]\right| = 0 \tag{27}$$

The frequency response function is used to obtain experimental eigenfrequencies. To solve the equation of motion Eq. (22), it is assumed that the solution exists of the form,

$$\{F(t)\} = \{F\}e^{i\omega t} \tag{28}$$

$$\{v(t)\} = \{v\}\, e^{i\omega t} \tag{29}$$

Substitution of these conditions into the equation of motion, this equation then becomes,

$$\left([K] - \omega^2\,[M]\right)\ \{v\}\, e^{i\omega t} = \{F\}\, e^{i\omega t} \tag{30}$$

$$\{v\} = \left([K] - \omega^2\,[M]\right)^{-1}\{F\} \tag{31}$$

The relationship between input (force excitation) and output (vibration response) of linear system is given by,

$$[H] = \frac{1}{[K] - \omega^2\,[M]} \tag{32}$$

where [H] is the frequency response functions matrix [22].

3. Identification of the structural parameters

The inverse method bases on the mathematical description and measurement of the natural frequencies of the structure with different cases. An added mass is used as a modification tool in this study. Firstly, the natural frequencies of the structure are measured at original and modified conditions. The dimensions of the structure are then identified by measuring real size. Secondly, the mathematical solution of the original structure is performed to obtain the ratio between the flexural rigidity EI and the mass per unit length m, but the actual values remain unknown. By using the mathematical solution of the modified structure, it is possible to evaluate directly either m or EI by using the previously identified EI/m ratio. A flowchart of this operation is given in Fig. 3.

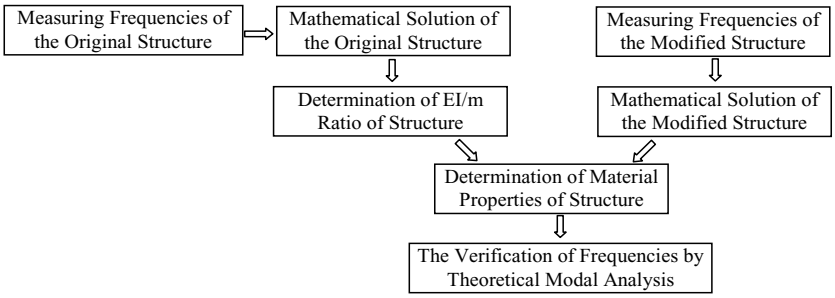


Fig. 3. General flowchart of parameter identification.

The method is superior to some known methods in two views. At first view, the added mass is easily removable after identification, and the original structure remains unchanged. The second view, only natural frequencies are measured. The second is very useful when performing measurements on large structures. Although the study is presented by a fixed end beam, the method is applicable to many structural systems that have mathematical descriptions.

3.1. Experimental measurements

A fixed end beam was selected in this study to show the efficiency of the proposed method. The beam is made of steel with the modulus of elasticity $E = 2 \times 10^{11}$Pa which is not measured but taken from the literature. The mass distribution per unit of length is obtained measured data as $m = 1.116$ kg/m. The profile is a rectangular structural with nominal size of 30 $\times$ 5 mm (Figs 4–5).

The shape and dimensions of the added mass used to as a modification tool are given in Figs 6–7. The mass of added body is measured as 1.835 kg and the mass moment of inertia of added body is calculated as 4.666 $\times$ 10^{-3} kgm^2.

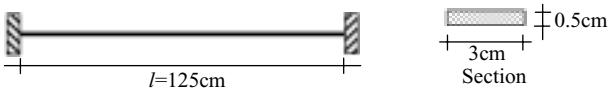


Fig. 4. The simplified fixed end beam model.



Fig. 5. The picture of the fixed end beam model.

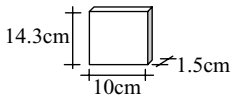


Fig. 6. The simplified figure of the added mass.



Fig. 7. The added mass used in the experimental modal analysis.

The single input single output method is used in the experimental measurement. The measurement equipments consist of an impact hammer Type B&K 8202, an accelerometer Type B&K 4382, and a signal analyzer unit Type B&K 3550. The test equipments are shown in Fig. 8. The structure is vibrated by using the impact hammer and the response of structure are measured by using the accelerometer. The impact and response signals are acquired in the time domain and then transformed into frequency domain. The natural frequencies are then derived from the frequency response functions.

The frequency response function measured on the original model is plotted in Fig. 9. In this figure, the x-axis values of the peaks are the frequencies of the model. The natural frequencies of the original model obtained from experimental modal analysis are given in Table 1.

The distances, where added mass is located, are selected by considering the mode shapes of the beam model. For this purpose, the theoretical and experimental modal analyses are named as follows (Table 2).

The results of the experimental modal analysis with added mass are given in Table 3.

Table 1
The first three natural frequencies of the model without added mass

The mode number	1	2	3
The natural frequencies (Hz)	16.125	44.75	88.125

Table 2
The analysis cases determined depending on the mode shape

Analysis case	The location of added mass
Case 1	Without mass
Case 2	$L_1 = L/2$
Case 3	$L_1 = L/3$
Case 4	$L_1 = L/4$

Table 3
The first three natural frequencies of the beam model with added mass

The natural frequencies (Hz)	1	2	3
Case 2	7.75	40.375	69.875
Case 3	8.75	32.50	73.50
Case 4	10.625	29.25	69.125

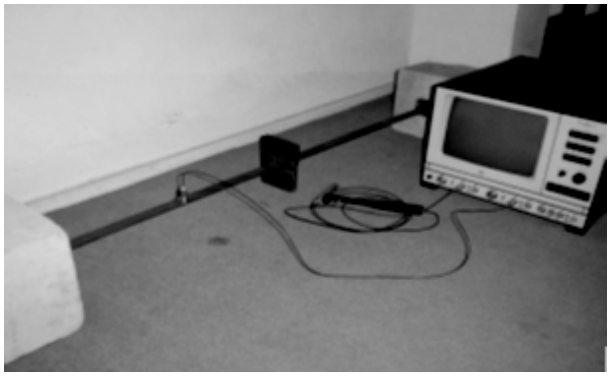


Fig. 8. Test equipments for the experimental modal analysis.

As expected, the eigenfrequencies of the modified beam with added mass is lower than the corresponding eigenfrequencies of the original beams.

3.2. Structural parameters

The first step of the parameter identification is the determination of averaged EI/m ratio. By using Eq. (13) which is the EI/m ratio of the original model, the averaged value are calculated from the first natural frequencies of the beam model and also presented in Table 4.

Table 4
The first three measured eigenfrequencies and related EI/m ratios

n	f (Hz)	$\omega = 2\pi f$ (rad/sn)	EI/m
1	16.125	101.3164	50.0674
2	44.750	281.1725	50.7509
3	88.125	553.7057	51.1987
		Average	**50.6723**

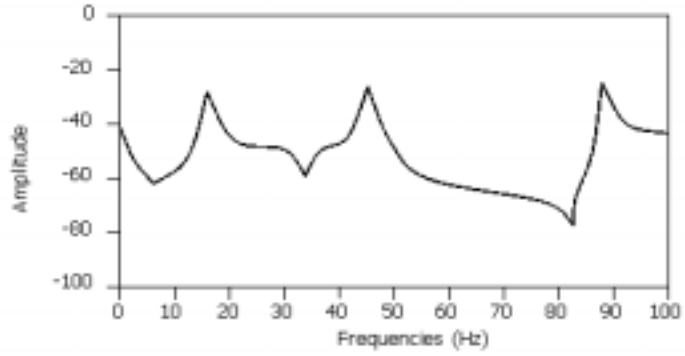


Fig. 9. The frequency response function measured on the original model.

Table 5
The identified mass distribution and elasticity modulus for Case 2

n	f (Hz)	ω (rad/sn)	λ	EI (Nm^2)		Mass(kg/m)	E (N/m^2)
1	7.7500	48.6947	2.61546	0.878373	56.934000	1.123572	$\mathbf{1.997 \times 10^{11}}$
2	40.3750	253.6836	5.96971	−176.0200	80.489100	1.588423	2.824×10^{11}
3	69.8750	439.0376	7.85341	8.754990	83.587500	1.649569	2.932×10^{11}

Table 6
The identified mass distribution and elasticity modulus for Case 3

n	f (Hz)	ω (rad/sn)	λ	EI (Nm^2)		Mass(kg/m)	E (N/m^2)
1	8.7500	54.9779	2.77908	0.995179	53.961900	1.064919	$\mathbf{1.892 \times 10^{11}}$
2	32.5000	204.2035	5.35598	−18.86740	59.054400	1.165417	2.071×10^{11}
3	73.5000	461.8141	8.05454	−284.3530	96.522900	1.904845	3.386×10^{11}

Table 7
The identified mass distribution and elasticity modulus for Case 4

n	f (Hz)	ω (rad/sn)	λ	EI (Nm^2)		Mass(kg/m)	E (N/m^2)
1	10.6250	66.7588	3.06240	1.235560	55.323300	1.091785	$\mathbf{1.940 \times 10^{11}}$
2	29.2500	183.7832	5.08113	0.891398	60.011900	1.134404	2.016×10^{11}
3	69.1250	434.3252	7.81115	−118.9330	86.898300	1.714907	3.048×10^{11}

It is obvious that the identified results vary within a range of $\pm 1.5\%$ around the average value and therefore the average value is used in further identification. The identified mass distribution and elasticity modulus for all cases are given in Tables 5–7.

It is seen from the above tables that the precisely measured first natural frequency is sufficient to determine the structural parameters by inverse method. Using the first identified results, the average values of mass distribution and elasticity modulus are calculated as 1.093425 and 1.943×10^{11}, respectively. It is obvious that the values identified using the first natural frequencies of the structure with added mass give acceptable results compared to the real value of the mass distribution and elasticity modulus.

3.3. Theoretical modal analysis

By taking into account the identified parameter by inverse method, the numerical modal analysis of the fixed steel beam is carried out to show the accuracy of parameters. The numerical modal analyses are performed SAP2000 [23] finite element program by using 1D beam elements. The analytical model of beam consists of 10 elements with two degrees of freedom.

The first three natural frequencies of the beam models obtained from theoretical modal analysis with and without added mass are shown in Table 8.

Table 8
The first three natural frequencies of the beam models

The natural frequencies (Hz)	1	2	3
Case 1	16.05	44.24	86.61
Case 2	7.65	38.26	67.72
Case 3	8.81	32.03	64.91
Case 4	10.59	28.74	63.70

Theoretical modal analyses results obtained using the identified parameters show clear similarity with the measured natural frequencies. They are evidence that the presented method can be easily and safely applied to the beams and similar structures.

4. Conclusion

The presented study is an application of the inverse method for parameter identification which is applied a fixed end beam. The method requires satisfactory measurements of the eigenfrequencies of the original and modified structure. The modified structure is established by using an added mass.

It is seen clearly from the results that all three solutions obtained with the first natural frequencies lie within a certain satisfactory range. The solutions obtained with higher frequencies in each case are not reliable because of the fact that the frequencies can not be measured accurately. Although executed with first eigenfrequencies it can be assumed that the result would show similar correctness by the execution of accurately measured higher eigenfrequencies.

The authors believe that the non-destructive method in this study can be developed in conjunction with satisfactory equipment and served as a useful tool even for more complex structures such as bridges.

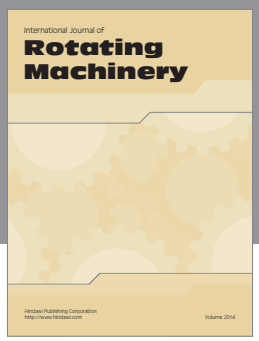
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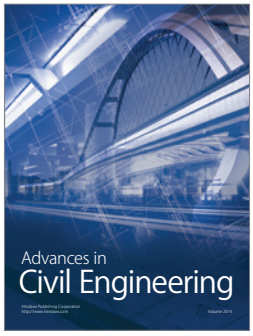


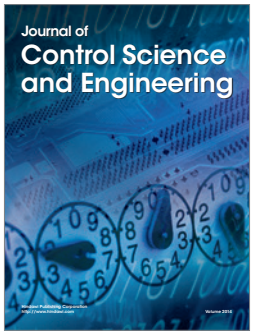






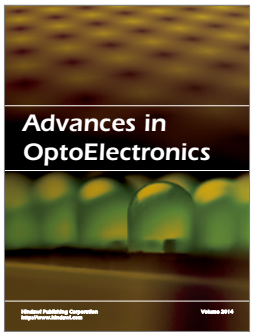






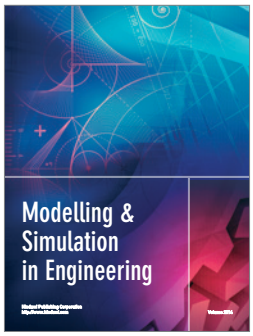






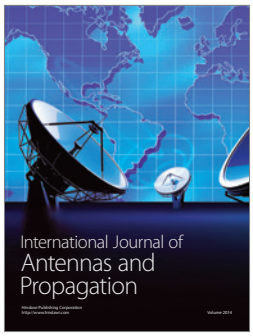




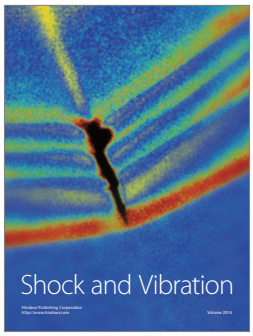


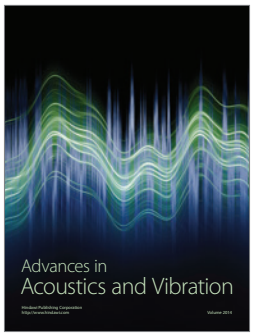












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